Problem of the Month
The Shape of Things

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

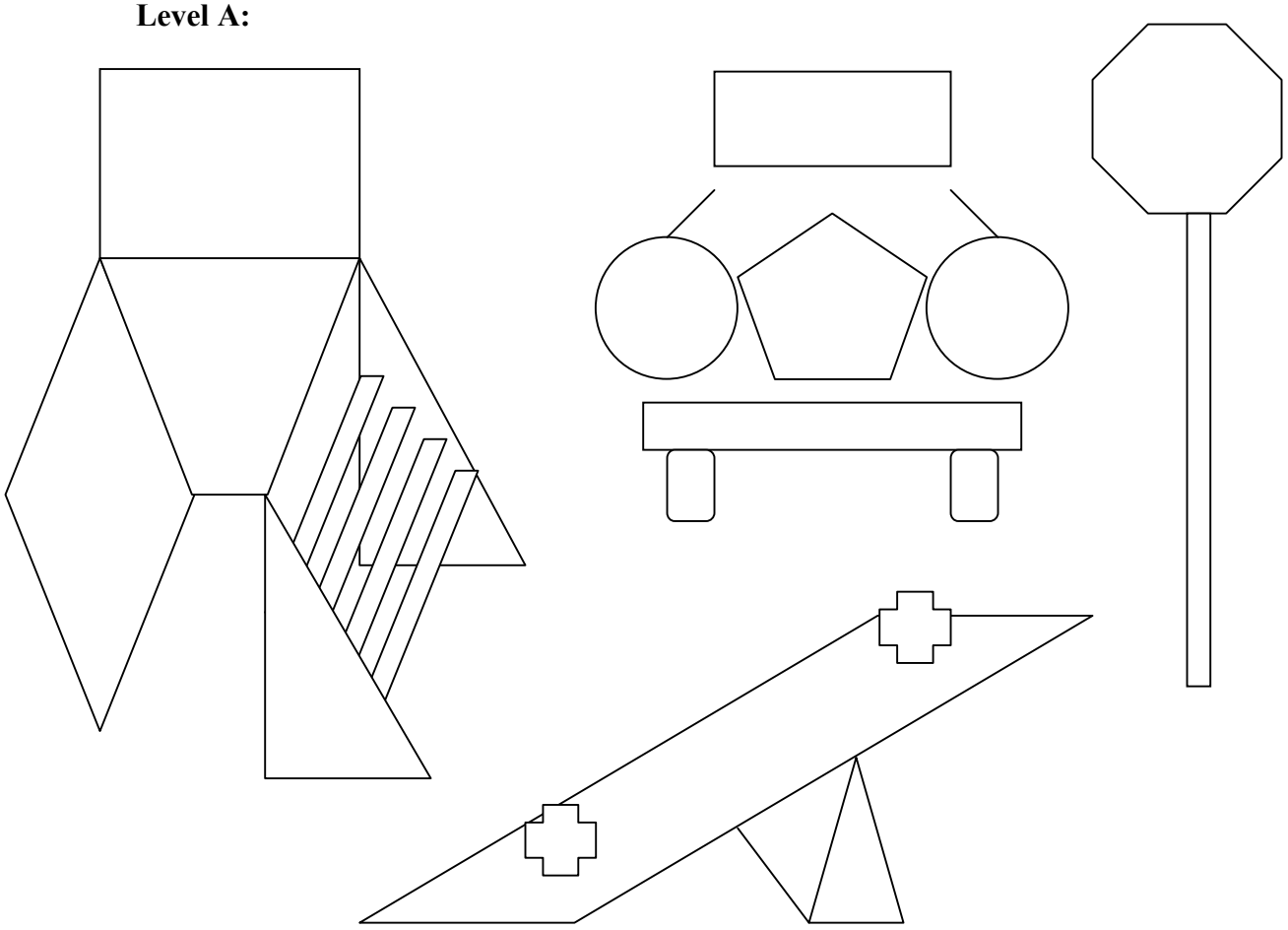
Overview:
In the Problem of the Month *The Shape of Things,* students use geometric reasoning to solve problems involving two-dimensional objects. The mathematical topics that underlie this POM are the attributes of polygons, symmetry, spatial visualization, mathematical justification, including inductive and deductive reasoning, and formal proof.

In the first level of the POM, students are presented with the task of examining, identifying, and comparing the attributes of the different geometric objects and describing their attributes. Level B requires students to use line symmetry to reason about a picture half folded from view. The students are asked to draw the whole picture, even through they see only half of the picture, but know the fold is a line of symmetry. In level C, students are given a set of corporate logos. Using their knowledge of rotational symmetry and spatial visualization, the students must determine which logos have rotational symmetry. Students then must design their own unique logo with rotational symmetry. In level D, the students investigate all the possible two-dimensional quadrilateral shaped kites given possible kit designs and relate each design to the diagonals' intersected position and angle as well as their two relative lengths. In level E, students are asked to formally prove or disprove a conjecture about the ways a regular hexagon can be divided into two equal size pieces.

The Problem of the Month

The Shape of Things

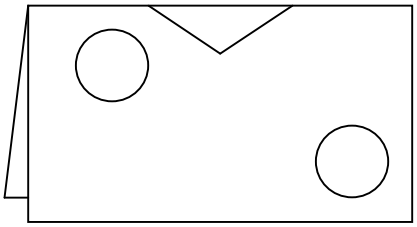
Level A:



Examine the scene of the playground above. There are a lot of different shapes that make up the toys and play-equipment. Name, draw and describe clearly all the shapes you can see in the playground picture.

Level B:

Laura and Jacob are drawing pictures to give to friends. Laura made a drawing using some shapes and a line of symmetry. She used the folded edge as the line of symmetry. Draw a picture of what the paper will look like when it is unfolded.



Jacob made a drawing with 5 shapes in all. When it was finished he noticed that

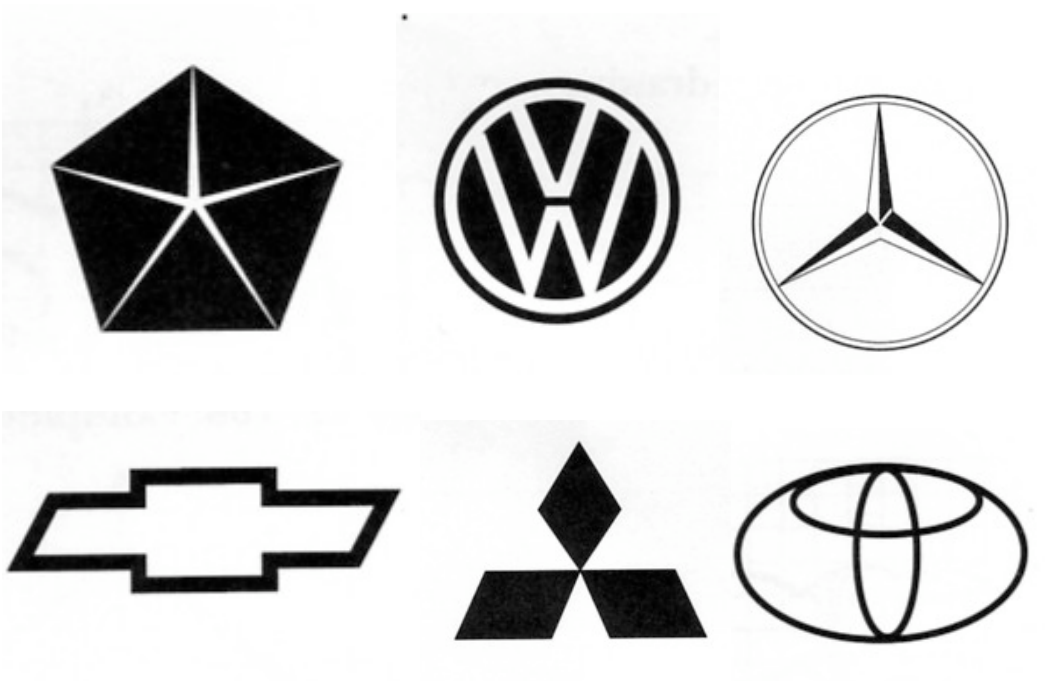
- his drawing had one line of symmetry
- he drew 4 shapes that had 4 corners
- he drew 1 shape that had 3 corners

Show what Jacob's drawing could have looked like.

Explain why you drew the drawing you did.

Level C:

You work for a company in its marketing department. It is your job to create a new company logo. You have heard that having a logo with rotational symmetry is attractive and useful for potential customers. You look through a page of company logos. Which of the logo below have rotational symmetry? Explain you findings.



Explain why a logo with rotational symmetry is attractive and useful for potential customers?

Create a new logo for your company that has rotational symmetry.

Illustrate all the line symmetries (if any) of your logo. Justify why your logo has rotational symmetry.

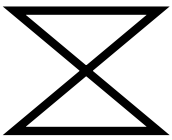
Level D:

Jake and Mark are making a kite. They have two sticks. "We need to the cut the sticks and lash them together," Mark said. "But, what shape will the kite be?" says Jake. "It depends on these sticks," Mark replied. What does Mark mean? Explore the types of kites you can make using two sticks. List the different types of kites you can make. Explain using geometry how the sticks affect the shapes of the kites.

Level E:
Laurie and Kristina wanted to share a cake equally. The cake was in the shape of a regular hexagon. Kristina said, "There are two ways to cut the cake to make equal size pieces. Either cut from one vertex to the opposite vertex or cut from the midpoint of one side to the midpoint of the opposite side. Those are the only two ways to make one straight cut and have two equal size cakes." Laurie said, "No, there are other ways to make one straight cut and share the cake equally besides your two ways." Who is right?

Prove your findings. If Kristina is correct prove why the two ways work and why there are only those two ways. If Laurie is right, state the method(s) for cutting the cake equally and prove your conjecture.

Explore other regular polygon shaped cakes. Does the method(s) used to cut regular hexagon cake work for cutting cakes of different regular shapes? State some conjectures and justify your findings.



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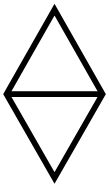


Primary Version Level A

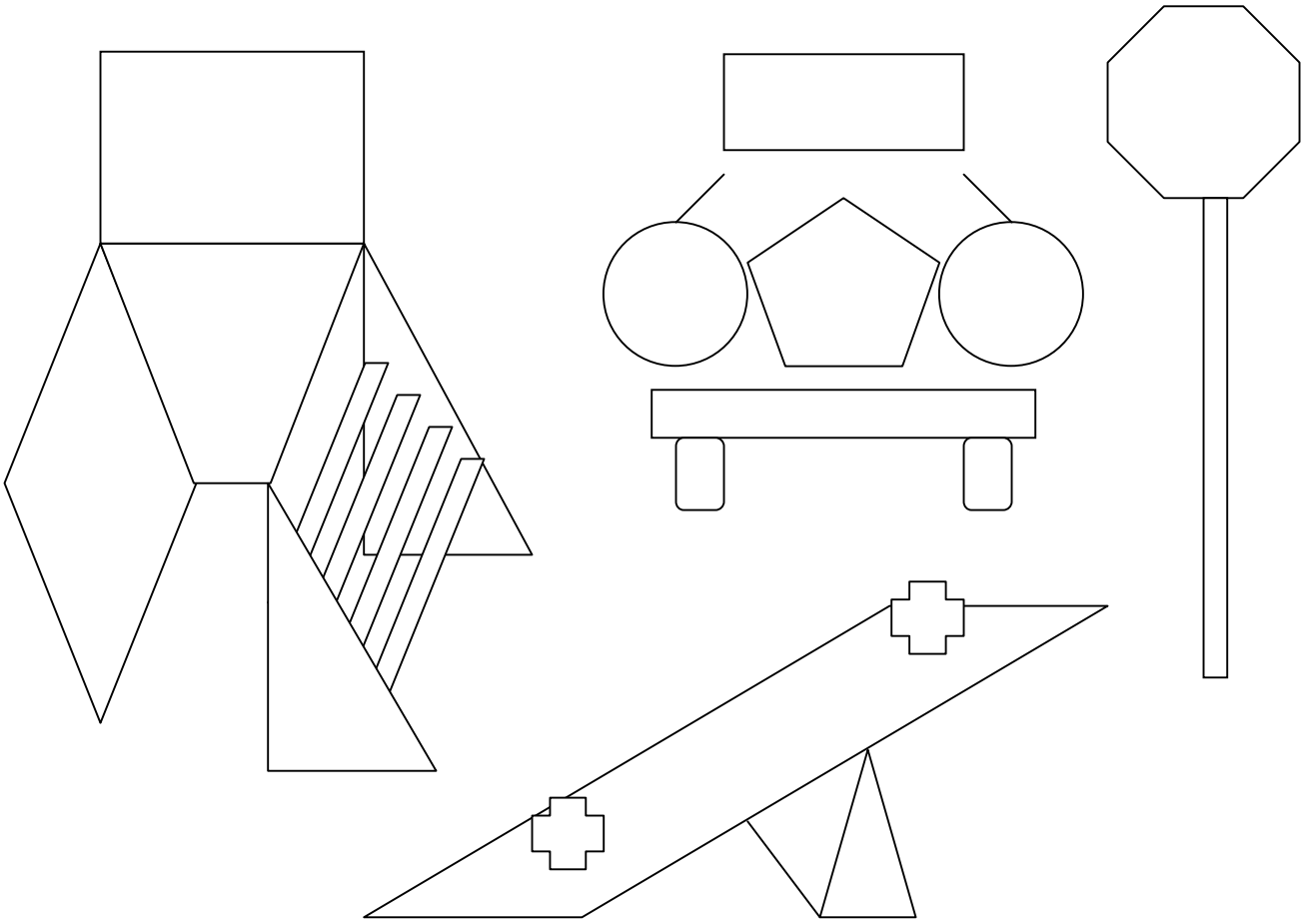
Materials: A scene of the playground. Access to individual shapes (cutouts). Papers and pencils to write or draw. Color crayons, markers or pencils.

Discussion on the rug: (Students have sets of some cut out shapes in front of them.) Teacher holds up the playground scene. "Here is a picture of a playground. What do you notice about the playground? What else do you notice about it?" (Teacher continues to ask children to notice different shapes in the play area. The teacher asks the students to find a shape among the ones cut out that is also in the picture. She asks the students to describe the shape.) "How can you tell the difference between that shape and another shape?" "Who knows the name of the shape?"

In small groups: (Each group has a playground scene, crayons, pencils or makers, and paper.)
(Teacher asks the students to find all the shapes in the picture.) Draw a picture of the shape you found. Where is the shape in the picture? What is the name of the shape? How do you know that shape, describe its parts.

At the end of the investigation: Students either discuss or dictate a response to this summary question. " Pick a shape in the playground. Draw a picture of the shape you found. Where is the shape in the picture? What is the name of the shape? How do you know that shape, describe its parts."

A Playground Scene



Problem of the Month

The Shape of Things

Task Description – Level A

This task challenges a student to identify and draw all the shapes seen in a drawing of playground equipment. Students must also describe the attributes of each shape.

Common Core State Standards Math - Content Standards

Geometry

Reason with shapes and their attributes.

1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g. color, orientation, overall size); build and draw shapes to possess defining attributes.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Indentify triangles, quadrilaterals, pentagons, hexagons and cubes.

3.G.1 Understand that shapes in different categories (e.g. rhombuses, rectangles, and others) may share attributes (e.g. having four sides), and that the shared attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Common Core State Standards Math – Standards of Mathematical Practice

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month

The Shape of Things

Task Description – Level B

This task challenges a student to use line of symmetry to reason about half of a folded picture and draw the missing half. Students are also given a set of conditions about a symmetrical drawing and asked to draw the shape.

Common Core State Standards Math - Content Standards

Geometry

Reason with shapes and their attributes.

1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g. color, orientation, overall size); build and draw shapes to possess defining attributes.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Indentify triangles, quadrilaterals, pentagons, hexagons and cubes.

3.G.1 Understand that shapes in different categories (e.g. rhombuses, rectangles, and others) may share attributes (e.g. having four sides), and that the shared attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

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Problem of the Month

The Shape of Things

Task Description – Level C

This task challenges a student to examine complex shapes to find which shapes have rotational symmetry. Students are also challenged to design their own complex shape with rotational symmetry. Students must draw in the lines of symmetry and develop a justification why each shape has rotational symmetry.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement.

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Understand congruence and similarity using physical models, transparencies, or geometric software.

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

High School – Geometry - Congruence

Experiment with transformations in the plane.

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

The Shape of Things

Task Description – Level D

This task challenges a student to investigate all the possible two-dimensional quadrilateral shaped kites that can be made given two sticks to make the diagonals of the kite. Students are asked to find all possible kite designs and relate each design to the diagonals' intersected position and angle as well as their two relative lengths.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement.

Geometry

Reason about shapes and their attributes.

3.G.1 Understand that shapes in different categories (e.g. rhombuses, rectangles, and others) may share attributes (e.g. having four sides), and that the shared attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Classify two-dimensional figures into categories based on their properties.

5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

High School – Geometry - Congruence

Prove geometric theorems.

G-CO.11 Prove theorems about parallelograms.

High School – Geometry – Modeling with Geometry

Apply geometric concepts in modeling situations.

G-MG.3 Apply geometric methods to solve design problems.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Problem of the Month

The Shape of Things

Task Description – Level E

This task challenges a student to prove or disprove a conjecture about the ways a regular hexagon can be divided into two equal size pieces.

Common Core State Standards Math - Content Standards

High School – Geometry – Congruence

Understand congruence in terms of rigid motions.

G-CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

G-C0.10 Prove theorems about triangles.

G-CO.11 Prove theorems about parallelograms.

Make geometric constructions.

G-CO.12 Make formal geometric constructions.

High School – Geometry – Circles

Understand and apply theorems about circles.

G-C.2 Indentify and describe relationships among inscribed angles, radii, and chords include the relationship between central inscribed and circumscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circles.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

The Shape of Things

Task Description – Primary Level

This task challenges a student to identify and draw all the shapes seen in a drawing of playground equipment. Students must also describe the attributes of each shape. Students use cut-outs and markers to help them draw and organize their thinking.

Common Core State Standards Math - Content Standards

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects.

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Analyze, compare, create, and compose shapes.

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g. number of sides and vertices/corners and other attributes (e.g. having sides of equal length.

Reason with shapes and their attributes.

1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g. color, orientation, overall size); build and draw shapes to possess defining attributes.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Indentify triangles, quadrilaterals, pentagons, hexagons and cubes.

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